# "Can you make/measure this asphere for me?"

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**Abstract:** The conventional characterization of an asphere's nominal shape is problematic: its coefficients are loaded with unnecessary digits and are unintelligible at first sight. There are related complications in design, fabrication, and testing. A recent solution is demonstrated for these shortcomings.

**OCIS codes:** (220.1250) Aspherics; (220.4830) Optical systems design; (220.4840) Optical testing; (220.4610) Optical fabrication.

## 1. Asphere characterization: problems and remedies

Fabricators —at both the system and component level— as well as designers and metrologists all obviously need standards and conventions for working on optical aspheres. Questions like those in the title are constantly asked within this community. These requests are presently hindered by a specification of shape that is increasingly inappropriate to the industry's evolving requirements. New technologies for generating, finishing, and testing aspheres continue to deliver advances that enable these surfaces to be made more cost-effectively, with stronger aspheric departures, and to higher precision. The industry is consequently positioned to benefit from more effective standards for specifying aspheric shape. This talk focuses on one such option for rotationally symmetric optics. The immediate benefits of these new characterizations include a roughly threefold reduction in the number of digits involved in specifying a modern asphere as well as far more direct measures of manufacturability.

After choosing the $z$ axis of a set of cylindrical polars, say $\{z,\rho,\theta\}$, to coincide with the optical axis, a part's shape can be specified explicitly as $z(\rho)$. Conventionally, $z(\rho)$ is expressed as a sum over $k$ of $A_k\,\rho^k$ added to the sag of a conic section [1]. Until recently, it was unusual for the polynomial part to hold more than two or three terms. By contrast, some examples of aspheres patented over the last five or six years are listed in Table 1. (Surprisingly, patents rarely specify aperture sizes, so I estimated those for the last three columns off their lens diagrams.) Tables like these are the basis of the exchanges discussed above. To allow for simple verification and error capture, a "lens print" is ideally also accompanied by a drawing and a sample of sag values. A significant risk of error remains, however, because these numbers say so little to the human eye and are repeatedly retyped by human fingers. It is also not immediately clear how many significant digits are required in the coefficients, curvatures, and conic constants. (Notice that they vary from 5 to 12 significant digits in Table 1, and it sometimes happens that an inadequate number of digits is specified by the designer when they initialize the fabrication chain).

**Table 1** Some recently patented aspheric optical surfaces. All lengths are in mm, and $A_k$ in units of $mm^{1-k}$.

| US Pat#: | 6,646,718 | 6,646,718 | 6,646,718 | 6,804,064 | 6,943,960 | 6,943,960 |
|---|---|---|---|---|---|---|
| Lens ID: | #L604 | #L616 | #L625 | #4 embod 1 | #2 embod 2 | #2 embod 3 |
| j | $A_{2+2j}$ (ord 4…16) | $A_{2+2j}$ (ord 4…16) | $A_{2+2j}$ (ord 4…16) | $A_{2+2j}$ (ord 4…18) | $A_{2+2j}$ (ord 4...20) | $A_{2+j}$ (ord 3...10) |
| 1 | -4.03355456E-07 | -2.83553693E-08 | -3.99248993E-10 | -2.86310E-04 | 2.5780279E-03 | 6.740893E-04 |
| 2 | 2.25776586E-11 | -1.12122261E-11 | 5.79276562E-14 | -5.00552E-06 | -3.1798526E-04 | 9.554210E-04 |
| 3 | -2.19259878E-14 | -2.05192812E-16 | 3.53241478E-18 | 4.18669E-07 | 4.6615960E-05 | -1.501880E-05 |
| 4 | 4.32573397E-18 | -1.55525080E-20 | -4.57872308E-23 | -2.46109E-08 | -5.2321154E-06 | 5.185462E-07 |
| 5 | -7.92477159E-22 | -4.77093112E-24 | -6.29695208E-27 | 6.13238E-10 | 4.3261277E-07 | 1.819851E-06 |
| 6 | 7.57618874E-26 | 8.39331135E-28 | 1.57844931E-31 | -1.79780E-12 | -2.4552167E-08 | -6.694942E-07 |
| 7 | -7.14962797E-30 | -8.97313681E-32 | -2.19266130E-36 | -2.00092E-13 | 8.9191847E-10 | 1.659582E-08 |
| 8 | | | | 2.66967E-15 | -1.8582400E-11 | 8.524903E-09 |
| 9 | | | | | 1.6845886E-13 | |
| **digit count:** | **76** | **76** | **77** | **60** | **84** | **64** |
| rad of curv: | -68.248613899 | 101.254238115 | -193.582989843 | 9.234 | 5.7054 | 4.6904 |
| conic const: | -1.3312 | 0.0 | 0.0 | 0.0 | 0.0 | -0.31638 |
| CA: | 110.334 | 143.652 | 268.202 | 10 | 8 | 8 |

The first three columns in Table 1 are from lithography systems while the others are from digital zoom cameras. This sample of exemplary patents from Zeiss, Ricoh, and Fujinon clarifies the trend towards more complex aspheric shapes. The main cause for trouble in these characterizations is that the individual terms in the polynomials largely cancel with one another when they are summed. In fact, individual terms can take values that are millions of times larger than their sum. That is why so many digits are needed in the coefficients and yet each term means nothing on its own. The number of digits in just the polynomial coefficients is given in the fourth last row. As discussed in this talk, the massive cancellation brings a variety of disadvantages. One effective solution is to use an orthogonalized basis, and two such bases were recently proposed for this application [2]. One of these expresses the surface as

$$z(\rho) = \frac{c_{\mathrm{bfs}}\,\rho^2}{1+\sqrt{1-c_{\mathrm{bfs}}^2\,\rho^2}} + \frac{s\,(1-s)}{\sqrt{1-c_{\mathrm{bfs}}^2\,\rho_{\max}^2\,s}} \sum_{m=0}^{M} a_m\, Q_m^{\mathrm{bfs}}(s), \tag{1}$$

where $c_{\mathrm{bfs}}$ is the curvature of the best-fit sphere, $\rho_{\max}$ is one half of the full aperture, $s = \rho^2 / \rho_{\max}^2$, and $Q_m^{\mathrm{bfs}}(s)$ is a polynomial of order $m$. When expressed in this way, the same surfaces of Table 1 are presented in Table 2.

**Table 2** The aspheres of Table 1 characterised by using Eq.(1). Each $a_m$ is given in units of nm.

| US Pat#: | 6,646,718 | 6,646,718 | 6,646,718 | 6,804,064 | 6,943,960 | 6,943,960 |
|---|---|---|---|---|---|---|
| Lens ID: | #L604 | #L616 | #L625 | #4 embod 1 | #2 embod 2 | #2 embod 3 |
| m | $a_m$ | $a_m$ | $a_m$ | $a_m$ | $a_m$ | $a_m$ |
| 0 | -1,279,170 | 2,252,133 | -349,932 | 175,721 | -21,119 | 172,996 |
| 1 | -14,021 | -177,695 | -6,442 | -1,420 | -41,170 | -63,085 |
| 2 | 3,461 | -32,347 | 46,321 | 384 | -1,381 | 3,677 |
| 3 | 6,672 | 2,293 | -6,940 | 119 | -3,577 | -438 |
| 4 | 1,260 | -211 | -591 | -29 | -1,027 | -354 |
| 5 | 320 | 1 | 74 | -71 | -345 | -47 |
| 6 | dropped | dropped | dropped | dropped | -147 | -102 |
| 7 | | | | | -62 | -58 |
| 8 | diff < 1nm | diff < 2nm | diff < 2nm | diff < 2nm | dropped | |
| **digit count:** | **27** | **26** | **24** | **20** | **30** | **28** |
| $1/c_{\mathrm{bfs}}$: | -72.322437 | 108.02985 | -194.49576 | 10.58561 | 5.046426 | 4.659623 |
| CA: | 110.334 | 143.652 | 268.202 | 10 | 8 | 8 |

The orthogonality of the polynomials used above means that the mean-square value of a linear combination of them is just the sum of the squares of the coefficients in that sum. These particular polynomials are orthogonalized with respect to the slope of the normal departure from the best-fit sphere. That is, this mean square slope is just the sum of the squared coefficients [2], and this corresponds directly to fringe density in an interferometric test. This also means that terms cannot cancel and, as a benefit, the same surfaces can be described to nm-level accuracy with typically three times fewer digits. The associated coefficients are like spectral components and, in stark contrast to those of Table 1, they can be toleranced and have direct interpretations. It becomes trivial to identify redundant coefficients that can be dropped and, even more importantly, it is straightforward to determine if an asphere can be tested by using a single full-aperture interferogram [3]. It has also been shown that estimates as to whether an asphere can be tested with stitched interferometry [4] can also be determined simply from these coefficients [5].

## 2. Conclusions

The ideas discussed in this talk are shown to greatly simplify the process of answering questions like those in the title. In fact, the oftentimes critical estimates of testability become so simple that they can be incorporated directly as efficient constraints within the design process itself. The changes can benefit all parties in the asphere chain of production and ultimately help to deliver systems with improved performance.

## 3. References

[1] D.S. Goodman, "Geometrical Optics" in *Handbook of optics,* M. Bass, ed. (Optical Society of America, McGraw Hill, 1995) Sec. 1.10.

[2] G.W. Forbes, "Shape specification for axially symmetric optical surfaces", Opt. Exp. **15**, 5218-5226 (2007).

[3] G.W. Forbes and C.P. Brophy, "Asphere, O Asphere, how shall we describe thee?", SPIE Proceedings **7100**, 710002-1-15 (2008).

[4] P. Murphy et al., "Subaperture stitching interferometry for testing mild aspheres", SPIE Proceedings **6293**, 62930J-1-10 (2006).

[5] G.W. Forbes and C.P. Brophy, "Designing cost-effective systems that incorporate high-precision aspheric optics", Optifab, TD06-25 (2009).